MAP3D VERIFICATION EXAMPLE 9

A circular tunnel in a Mohr-Coulomb medium with an overlying fault

1 Description

This example involves calculating the stresses and displacements on a fault overlying a 5 m diameter cylindrical tunnel subjected to a hydrostatic stress field.

The model input parameters are based on an example from the BITEMJ User's Manual (1983). A version of this example also appears in the Rocscience Phase2 Tutorial Manual (2001). The problem is also detailed in Brady and Brown (2005) as shown in their Figure 1.

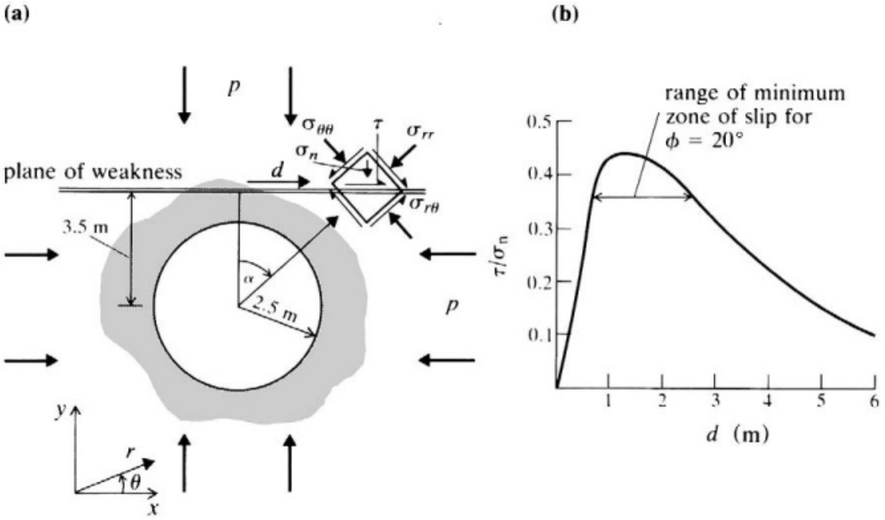

Figure 1 Problem description *(Figure 7.1 p208 from Brady and Brown (2005)).*

Using the equations for stresses around a circular hole in a hydrostatic stressfield and transformation equations gives the normal and shear stresses on the fault plane as:

$$
\begin{aligned}
\sigma_n &= \frac{1}{2}(\sigma_{rr}+\sigma_{\theta\theta})+\frac{1}{2}(\sigma_{rr}-\sigma_{\theta\theta})\cos 2\alpha \\
&= p\left(1-\frac{a^2}{r^2}\cos 2\alpha\right) \\
\tau &= \sigma_{r\theta}\cos 2\alpha-\frac{1}{2}(\sigma_{rr}-\sigma_{\theta\theta})\sin 2\alpha \\
&= p\frac{a^2}{r^2}\sin 2\alpha
\end{aligned}
$$

Note that it is also possible to output the predicted fault in-plane components such as shear and normal stresses directly from Map3D (see discussion later).

The MAP3D model geometry is shown in Figure 2 with tunnel (white); the overlying fault (yellow); plastic material zone (green) and the solution grid located at the mid-length of the cylindrical tunnel.

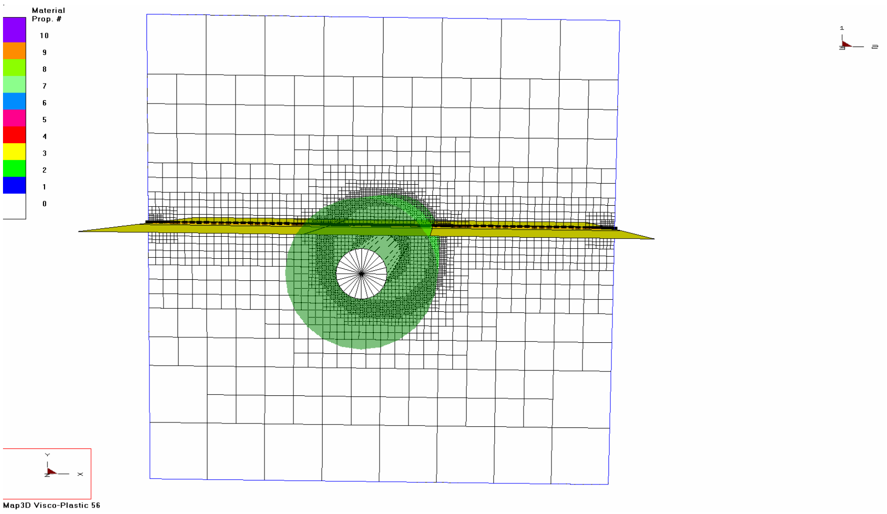

Figure 2: The MAP3D 5m diameter tunnel with overlying fault model.

The 3D cylindrical tunnel is defined by fictitious force elements. The radius of 2.5 m is relatively small compared to the overall length of the tunnel. This provides solutions that are equivalent to a 2D plane strain condition. The fault, built with displacement discontinuity elements, is located 1 m above the tunnel crown (yellow). The plastic material zone is also constructed from fictitious force element blocks as shown in Figure 2 (green volume). The zone has a radius of 7.5 m to encompass the yield zone produced by the nominated rock strength.

Figure 3 is a plan view with the plastic zone translucent so that the internal nodes that are concentrated near the solution grid can be seen. The section of fault passing through the volume is hidden. Figure 4 is an oblique view.

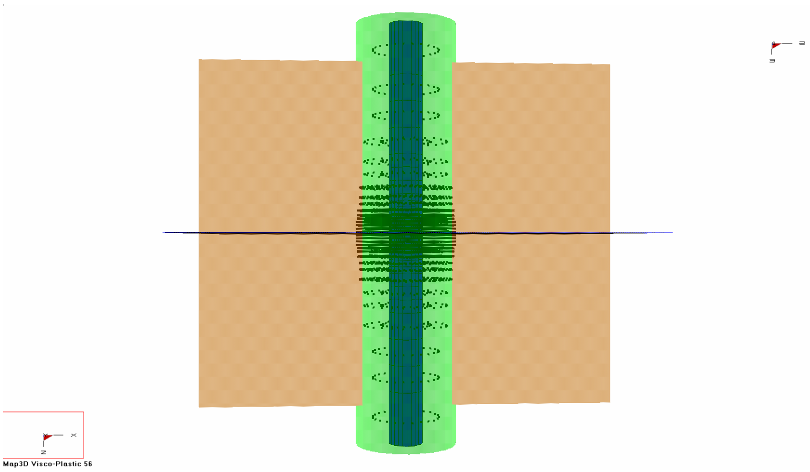

Figure 3: Plan view of the model showing internal nodes in the plastic zone.

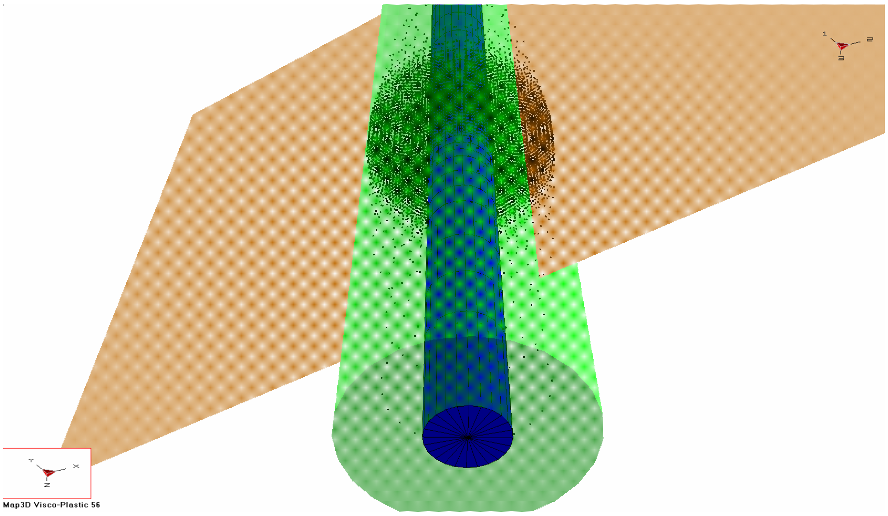

Figure 4: An oblique view of the model showing internal nodes in the plastic zone clustered around the solution grid location (hidden).

The host elastic material is defined as being isotropic and with a Young's modulus of 1,000 MPa and a Poisson's ratio of 0.20.

The plastic material zone is assumed to be linearly elastic, perfectly plastic, with a failure surface defined by the Mohr-Coulomb criterion with cohesion of 1.15 MPa and a friction angle of 30° for a UCS of 4.0 MPa. The analyses use a dilation angle of 0° (non-associated flow rule case).

The Viscous Modulus (Gs) is set to 100 MPa or 10% of the Young's modulus. The Gs helps in the program solution.

The model has a constant hydrostatic compressive in-situ stress field of 10 MPa. ($P_1 = P_2$). It is also assumed that there is no internal pressure inside the hole.

2.1 Results – Elastic Medium

The first solution is for an elastic medium so the plastic material zone is made inactive in this analysis. This analysis allows a direct comparison with the elastic medium analytical solution and other numerical modeling programs such as BITEMJ.

The shear stress/normal stress ratio from the analytical solution shows that slip on the fault will not occur for friction angles > 24°. Slip will occur for lower values and the following analyses all use a friction angle of 20°.

The ratio of shear stress over normal stress is shown in Figure 4 and matches the possible predicted 'slip' shear zone extent as described earlier.

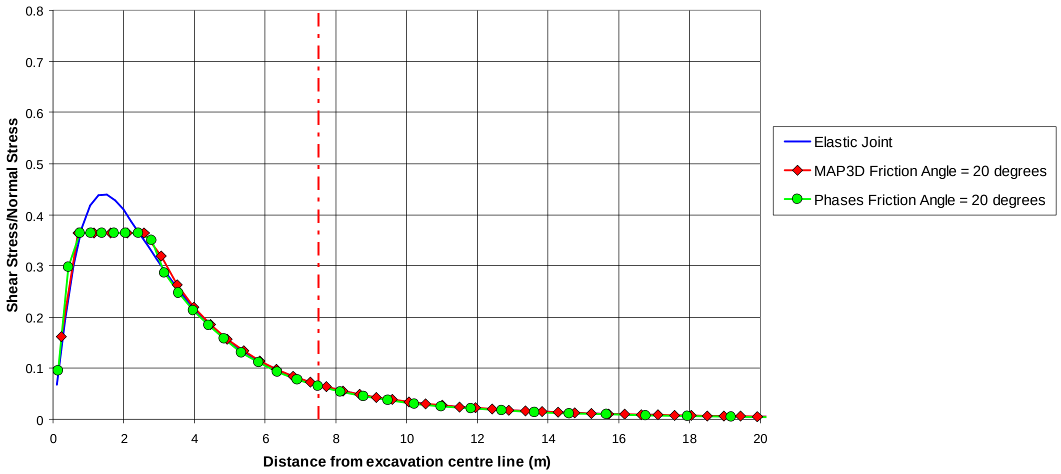

Figure 4: Ratio of shear to normal stress on the fault plane (friction angle = 20°).

In Map3D it is possible to contour in-plane stress and strain components. Simply check the User Manual '*In-plane'* i.e. Plot>Stress> Tip (In-plane shear stress).

The dashed red line indicates the 7.5 m radius of the plastic zone used in the following Mohr-Coulomb medium analysis.

2.2 Results - Mohr-Coulomb medium

This solution is for a Mohr-Coulomb medium with a UCS of 4 MPa.
The first analysis was without the fault plane to verify the plastic solution.
The radial and hoop stresses at a section mid tunnel height is shown in Figure 5
The yield zone extends to a radius of 4.33 m's.

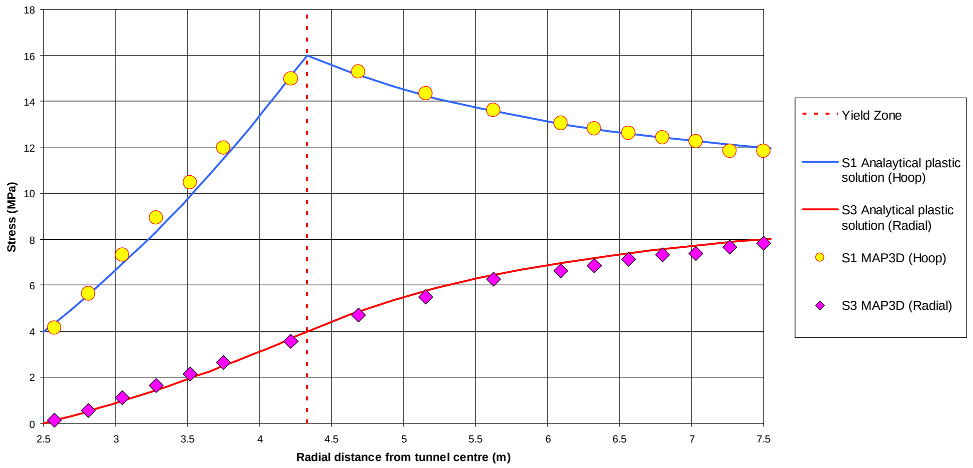

Figure 5: Radial σ_r (S_3) and tangential stresses $\sigma_{\theta.}$(S_1) plotted versus radius.

Figure 6 shows the minor (S_3) and major principal stresses (S_1) plotted on line grids at the mid-tunnel height and the fault horizon.

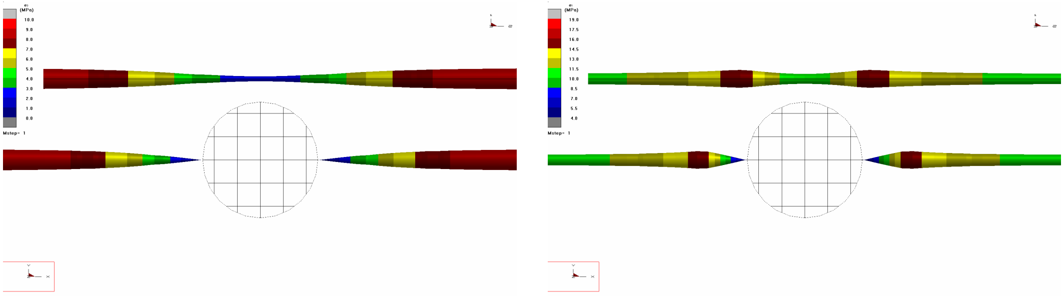

Figure 6: Minor (S_3) and Major Principal Stresses. (S_1) on line grids.

Figure 7 shows contours of plastic Major Principal Strain (E1) with an overlying 1 m grid. Strain values are also shown on a line grid at the fault horizon.

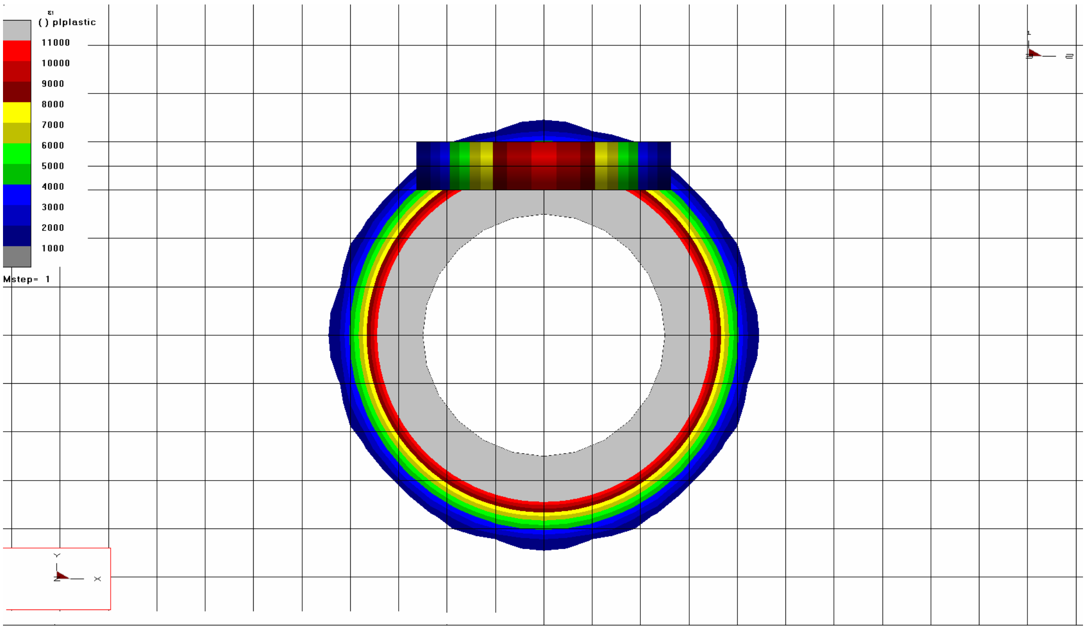

Figure 7: Contours of plastic Major Principal Strain (E1).

The plastic material zone is at a radius of 7.5 m and so beyond the 4.33 m radius yield zone. Note that the fault horizon is within the yield zone above the tunnel.

The fault modeled with the DD (Displacement Discontinuity) elements was added and the result is shown in Figure 8. Figure 9 shows a combined plot of the results for both the elastic and Mohr Coulomb mediums.

There are alternative solutions for any nonlinear analysis depending on the discretization used by the model. A similar discretization was used for both the Phase2 and Map3D analyses used in the following comparison plots.

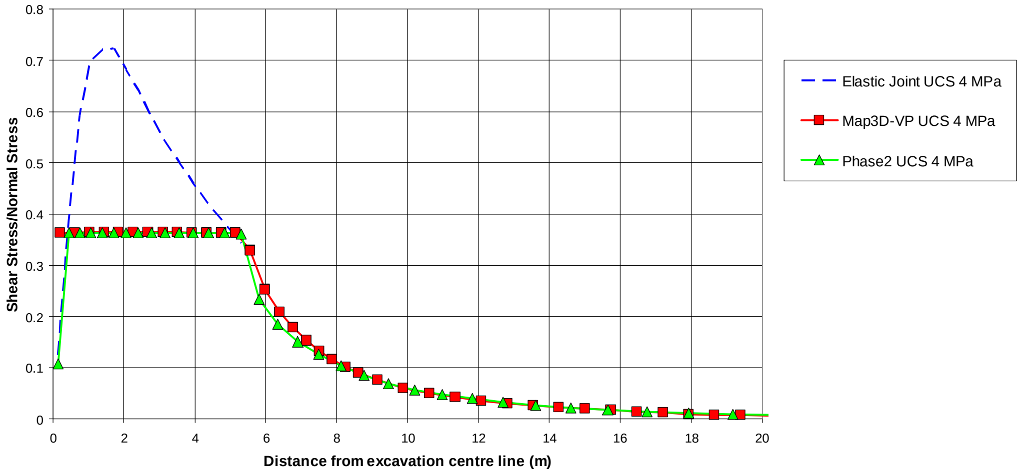

Figure 8: Ratio of shear/normal stress on the fault plane: UCS = 4 MPa medium.

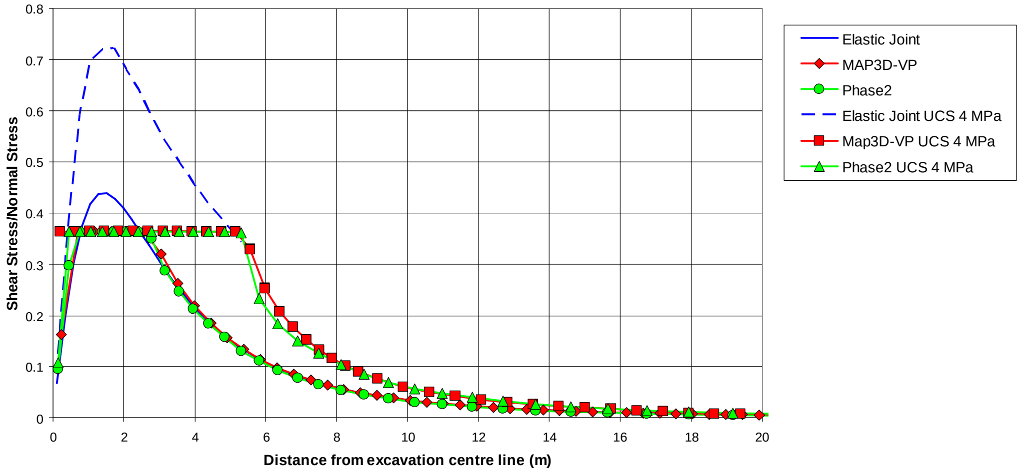

Figure 9: Ratio of shear/normal stress with Elastic and UCS = 4 MPa mediums.

The various Map3D results are in agreement with the equivalent analytical and numerical solutions.

The plot shows that the slip zone has approximately doubled for the Mohr Coulomb medium case from 2.5 to 5.2 m's.

The following Figures 10 to 12 show for the Mohr Coulomb medium the influence that the yielding fault has on the major and minor principal stresses $\boldsymbol{\sigma_1}$ and $\boldsymbol{\sigma_3}$, and on total displacements $\boldsymbol{u_t}$.

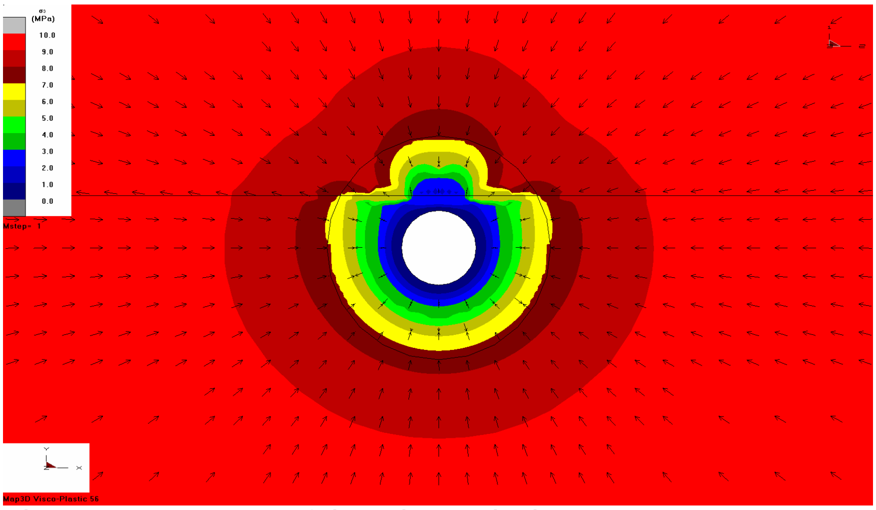

Figure 10: Contours of the Minor Principal Stresses σ3

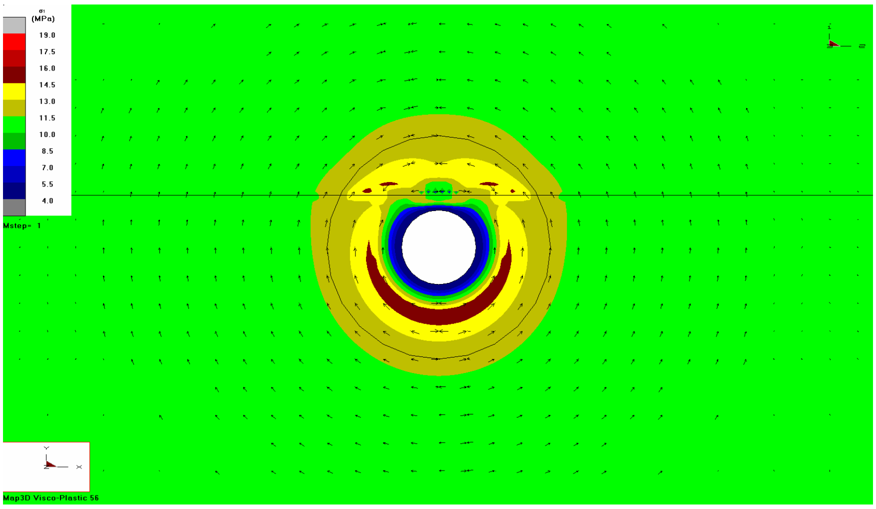

Figure 11: Contours of the Major Principal Stresses σ1

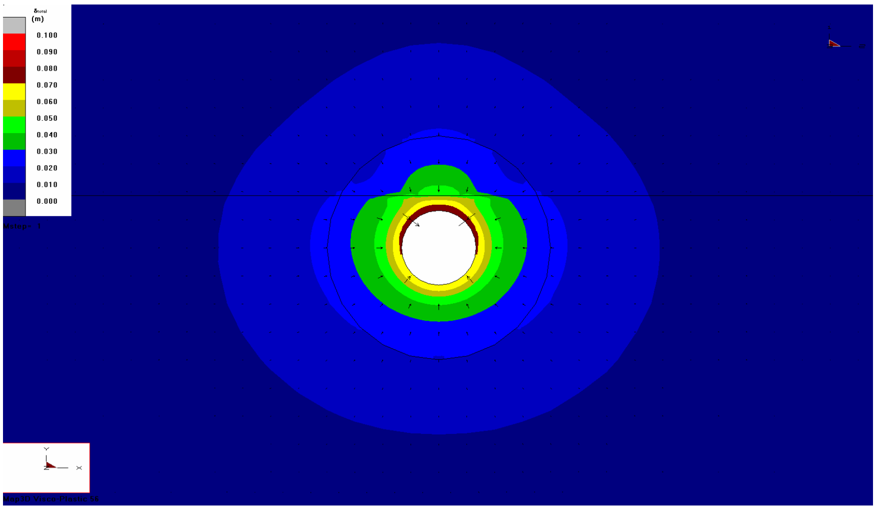

Figure 12: Contours of Total displacement u_t.

3 Applications

A simple extension of this example to a typical real 3D problem is a shaft intersected by a fault. Figure 13 shows a fault (yellow) intersecting near the base of a 6 m diameter shaft. The plastic material zone is show as green

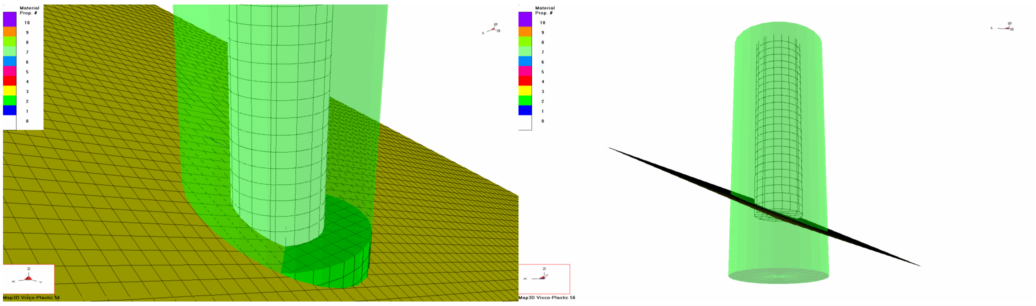

Figure 13: A fault intersecting a shaft model. .

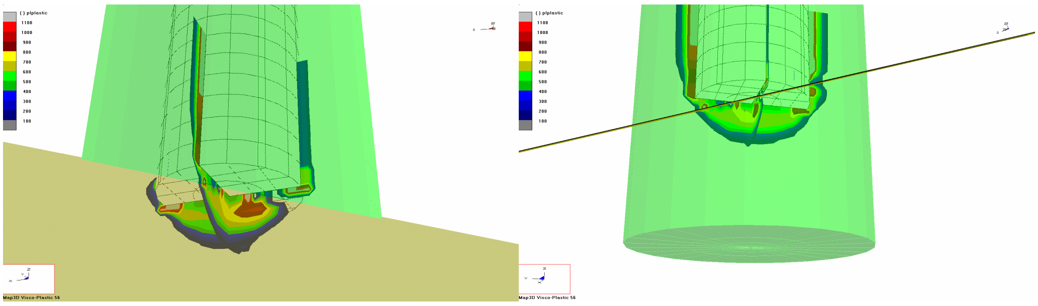

Figure 14: Plastic strain in the rock at the fault intersection location.

4 References

Crotty, J.M. (1983) *User's Manual for BITEMJ – Two-dimensional Stress Analysis for Piecewise Homogeneous Solids with Structural Discontinuities.* Geomechanics Computer Program No 5. CSIRO Division of Geomechanics: Melbourne.

Brady, B.H.G. and Brown, E.T., *Rock Mechanics for Underground Mining*, Springer Science, 3rd Edition 2005, pp204-208.

Rocscience Phase2 Tutorial manual (2001).

5 MAP3D Input Files

Tunnel with overlying fault le no VP zone.inp: has elastic fault in elastic medium.

Tunnel with overlying fault MC 3.inp: has elasto-plastic fault and MC plastic zone.